**Sequences and Series 4**

**1.** By using the difference method find the sim of the first n terms:

 Let

**2.** Express in partial fractions. Hence, find a simple expression for and determine whether S converges when n tends to infinity.

**3.** If , find the value of .

 Deduce the general term and hence find .

**4.** The sum of the first 2n terms of a series . Find the sum of the first n terms and the nth term of this series. Show that this series is an arithmetic.

 Hence .

 which is a constant.

 Therefore, this series is an arithmetic with a common different .

**5.** In a set of integers between the numbers 1 and 10,000,

 **(a)** how many of these numbers are divisible by 3,4, 5 and 11?

 **(b)** how many of these numbers are divisible by 3,4, 5 or 11?

 **(a)** The number must be divisible by the least common multiple of 3,4, 5 and 11, which is 660.

 Denote be the greatest integer smaller or equal to x .

 Then the number between the numbers 1 and 10,000 which are divisible by 3,4, 5 and 11

 **(b)** Let A = { integers from 1 through 10000 that are multiples of 3 }

 B = { integers from 1 through 10000 that are multiples of 4 }

 C = { integers from 1 through 10000 that are multiples of 5 }

 D= { integers from 1 through 10000 that are multiples of 11 }

 Denote be the greatest integer smaller or equal to x . We have :

 ,, ,

 By the Principle of Exclusion and Inclusion,

 Required number

 =

**6.** The sum of the first n terms of a geometric sequence is given by . Find

 **(a)** the nth term,

 **(b)** the common ratio,

 **(c)** the smallest value of n such that .

 **(a)**

 **(b)**

 **(c)**

**7.** Verify the identity . Hence, using the method of difference, prove that

 .

 Deduce the sum of the infinite series

 where

**8.** If are three consecutive terms in an arithmetic progression.

Show that also from three consecutive terms in an arithmetic progression.

 Since are in A.P.

 therefore

 Therefore also in A.P.

**9.** If S is the sum of the series , for , by considering

 , or otherwise, show that .

 **Method 1**

 **Method 2**

 Replace

 **Method 3**

**10. (a)** The sequence of positive integers is grouped into four as follows:

 Show that the sum of all integers in the kth bracket is .

**(b)** If the integers are similarly grouped with m integers in each bracket, find the sum of all integers in the nth bracket in terms of m and n.

 Hence, show that are in arithmetic progression.

**(a)** Last term in the kth bracket is .

 First term in the kth bracket is .

**(b)** The sequence is

 Last term in the nth bracket is .

 First term in the nth bracket is .

 Since , hence are in arithmetic progression.

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